

What's Wrong with Pauli-Villars Regularization in QED_3 ?

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Abstract

In this letter we argue that there is no ambiguity between the Pauli-Villars and other methods of regularization in (2+1)-dimensional quantum electrodynamics with respect to dynamical mass generation, provided we properly choose the couplings for the regulators.

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It is well known that gauge theories in (2+1)-dimensional space-time^[1], though super-renormalizable, show up inconsistencies already at one loop, arising from the regularization procedure adopted to evaluate ultraviolet divergent amplitudes such as the photon self-energy in spinor quantum electrodynamics. In the latter, if we use analytic^[2] or dimensional^[3] regularization, the photon is induced a topological mass, in contrast with the result obtained through the Pauli-Villars^[1] scheme, where the photon remains massless when we let the auxiliary masses go to infinity.

Recently, alternative constructs^{[4],[5]} have been proposed in order to clarify this matter, which essentially rely upon using causal dispersion relations. They put forward that the photon indeed dynamically acquires a topological

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mass. Thus, we are led to ask whether the ordinary Pauli-Villars prescription shouldn't be taken with a grain of salt.

Let us begin by close examining the conditions that must be imposed on the masses and coupling constants of the regulator fields such that a regularized closed fermion loop in 2+1 dimensions is rendered finite along the calculations. Consider the integral corresponding to a fermion loop with n vertices to which we associate n external photon lines with momenta k_i ($i=1,2,\dots,n$). This integral is proportional to

$$\int d^3p \frac{\text{Tr}[\gamma_{\mu_1}(m + \not{p})\gamma_{\mu_2}(m + \not{p} + \not{k}_1)\dots\gamma_{\mu_n}(m + \not{p} + \dots + \not{k}_{n-1})]}{(m^2 - p^2 + i\epsilon)[m^2 - (p + k_1)^2 + i\epsilon]\dots[m^2 - (p + \dots + k_{n-1})^2 + i\epsilon]} \quad (1)$$

so, for large p , its integrand behaves like p^{-n} whereas for $n < 4$ the integral diverges as

$$\int_0^\infty \frac{p^2 dp}{p^n} \sim \int_0^\infty \frac{dp}{p^{n-2}} .$$

This integrand can be written as

$$\mathcal{I} \equiv \frac{P_n(p) + mP_{n-1}(p) + m^2P_{n-2}(p) + \dots + m^n}{P_{2n}(p) + m^2P_{2n-2}(p) + \dots + m^{2n}} , \quad (2)$$

where $P_i(p)$ stands for a polynomial of degree i in the components of p . We can write the denominator of \mathcal{I} in the form

$$P_{2n}(p) \left(1 + m^2 \frac{P_{2n-2}(p)}{P_{2n}(p)} + \dots + m^{2n} \frac{1}{P_{2n}(p)} \right)$$

and, for large p , perform the expansion

$$\frac{1}{\left(1 + m^2 \frac{P_{2n-2}(p)}{P_{2n}(p)} + \dots + m^{2n} \frac{1}{P_{2n}(p)} \right)} \sim 1 - m^2 \frac{P_{2n-2}(p)}{P_{2n}(p)} + \dots ,$$

so that the integrand \mathcal{I} behaves like

$$\mathcal{I} \sim \frac{P_n}{P_{2n}} - m^2 \frac{P_n P_{2n-2}}{P_{2n} P_{2n}} + m \frac{P_{n-1}}{P_{2n}} - m^3 \frac{P_{n-1} P_{2n-2}}{P_{2n} P_{2n}} + m^2 \frac{P_{n-2}}{P_{2n}} + \dots ,$$

i.e.,

$$\begin{aligned}
\mathcal{I} &\sim \frac{P_n}{P_{2n}} + m \frac{P_{n-1}}{P_{2n}} + m^2 \frac{P_n}{P_{2n}} \left[\frac{P_{n-2}}{P_n} - \frac{P_{2n-2}}{P_{2n}} \right] + \\
&\quad m^3 \frac{P_{n-1}}{P_{2n}} \left[\frac{P_{n-3}}{P_{n-1}} - \frac{P_{2n-2}}{P_{2n}} \right] + \dots \\
&= \sum_k m^k a_{-(n+k)}(p) ,
\end{aligned} \tag{3}$$

where

$$a_{-(n+k)}(p) \sim p^{-(n+k)} .$$

Therefore, in making the substitution

$$\mathcal{I}(m) \rightarrow \sum_i^{n_f} c_i \mathcal{I}(M_i) ,$$

where n_f is the number of auxiliary fermion fields, we must impose in the vacuum polarization case ($n = 2$) the following conditions:

$$\sum_i^{n_f} c_i = 0 , \tag{4}$$

$$\sum_i^{n_f} c_i M_i = 0 , \tag{5}$$

in order to eliminate the linear and logarithmic divergences, respectively.

Having settled down the basis for the Pauli-Villars regularization method, we turn to the calculation of the vacuum polarization tensor in spinor QED_3 . In the standard notation, the regularized expression for the vacuum polarization tensor reads

$$\Pi_{\mu\nu}^M(k) = \frac{ie^2}{(2\pi)^3} \sum_{i=0}^{n_f} c_i \int d^3p \frac{P(M_i)}{(M_i^2 - p_1^2)(M_i^2 - p_2^2)} , \tag{6}$$

where

$$c_0 = 1 \quad , \quad M_0 = m \quad , \quad M_i = m\lambda_i \quad (i = 1, \dots, n_f) , \tag{7}$$

$$p_{1,2} = p \mp \frac{1}{2}k \tag{8}$$

and

$$\begin{aligned} P(M_i) &= \text{Tr}\{\gamma_\mu(\not{p}_1 + M_i)\gamma_\nu(\not{p}_2 + M_i)\} \\ &= 2[M_i^2 g_{\mu\nu} + p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu} - g_{\mu\nu}(p_1 \cdot p_2) - iM_i \epsilon_{\mu\nu\alpha} k^\alpha] . \end{aligned} \quad (9)$$

For simplicity, but without loss of generality, we may choose both the electron mass and those of the auxiliary fields to be positive; the coefficients λ_i ultimately go to infinity to recover the original theory. Using the Feynman parametrization

$$\frac{1}{(M_i^2 - p_1^2)(M_i^2 - p_2^2)} = \int_0^1 d\xi \frac{1}{[M_i^2 - p_1^2 - (p_2^2 - p_1^2)\xi]^2} \quad (10)$$

and performing the momentum shift $p_\mu \rightarrow p_\mu + (\frac{1}{2} - \xi)k_\mu$, we get

$$\Pi_{\mu\nu}^M(k) = (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\Pi_1^M(k^2) + im\epsilon_{\mu\nu\alpha}k^\alpha\Pi_2^M(k^2) + \Pi_{GB}^M(k^2) , \quad (11)$$

where

$$\Pi_1^M(k^2) \equiv 4ie^2k^2 \sum_{i=0}^{n_f} c_i \int_0^1 d\xi \xi(1-\xi) \int \frac{d^3p}{(2\pi)^3} \frac{1}{(Q_i^2 - p^2)^2} , \quad (12)$$

$$\Pi_2^M(k^2) \equiv -\frac{2ie^2}{m} \sum_{i=0}^{n_f} c_i M_i \int_0^1 d\xi \int \frac{d^3p}{(2\pi)^3} \frac{1}{(Q_i^2 - p^2)^2} , \quad (13)$$

$$\Pi_{GB}^M(k^2) \equiv \frac{2}{3}ie^2g_{\mu\nu} \sum_{i=0}^{n_f} c_i \{I_i^1 + I_i^2\} , \quad (14)$$

with the following definitions

$$I_i^1 \equiv 3 \int_0^1 d\xi \int \frac{d^3p}{(2\pi)^3} \frac{1}{(Q_i^2 - p^2)} , \quad (15)$$

$$I_i^2 \equiv 2 \int_0^1 d\xi \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{(Q_i^2 - p^2)^2} \quad (16)$$

and

$$Q_i^2 \equiv M_i^2 - \xi(1-\xi)k^2 . \quad (17)$$

If we carry out the momentum integrations in (15) and (16) it is straightforward to arrive at $\Pi_{GB}^M(k^2) \equiv 0$, as expected by gauge invariance. Here

comes the crucial point: we can't blindly take only one auxiliary field with $M = \lambda m$ as usual; this choice is misleading for the subsidiary conditions (4) and (5) must be matched. This is possible only fixing $\lambda = 1$. Thus, the number of regulators must be at least two, otherwise we can't get the coefficients λ_i becoming arbitrarily large.

So, let us take

$$c_1 = \alpha - 1 \quad , \quad c_2 = -\alpha \quad , \quad c_j = 0 \quad ; \quad j > 2 \quad , \quad (18)$$

where the parameter α can assume any real value except zero and the unity, so that condition (4) is satisfied. For $\lambda_1, \lambda_2 \rightarrow \infty$,

$$\Pi_1^M(k^2) \rightarrow \Pi^{(1)}(k^2) = -\frac{e^2 k^2}{(2\pi)} \int_0^1 d\xi \, \xi (1 - \xi) \frac{1}{(M^2)^{\frac{1}{2}}} \quad , \quad (19)$$

where

$$M^2 \equiv m^2 - \xi (1 - \xi) k^2 \quad , \quad (20)$$

and, consequently, $\Pi^{(1)}(0) = 0$.

From (13) we find

$$\begin{aligned} \Pi_2^M(k^2) = \frac{e^2}{4\pi m} \int_0^1 d\xi \left\{ \frac{m}{[m^2 - \xi (1 - \xi) k^2]^{\frac{1}{2}}} + \frac{(\alpha - 1)M_1}{[M_1^2 - \xi (1 - \xi) k^2]^{\frac{1}{2}}} \right. \\ \left. - \frac{\alpha M_2}{[M_2^2 - \xi (1 - \xi) k^2]^{\frac{1}{2}}} \right\} \quad . \quad (21) \end{aligned}$$

Taking the limit $\lambda_1, \lambda_2 \rightarrow \infty$ for $k = 0$, yields

$$\Pi^{(2)}(0) = \frac{\alpha e^2}{4\pi m} (1 - s) \quad , \quad (22)$$

$$s \equiv \text{sign}(1 - \alpha^{-1}) \quad . \quad (23)$$

For $0 < \alpha < 1$, which corresponds to $s = -1$ and couplings c_1 and c_2 having the same sign, $\Pi^{(2)}(0) \neq 0$; in this case the photon acquires a topological mass, proportional to $\Pi^{(2)}(0)$, coming from proper insertions of the antisymmetric sector of the vacuum polarization tensor in the free photon propagator. If we assume that α is outside this range, $s = 1$, c_1 and c_2 have opposite signs and $\Pi^{(2)}(0)$ vanishes. We then conclude that this arbitrariness

in the choice of the parameter α reflects in different values for the photon mass. The new parameter s may be identified with the winding number of homotopically nontrivial gauge transformations and also appears in lattice regularization^[6].

Now we face another problem: which value of α leads to the correct photon mass? A glance at equation (13) and we realize that $\Pi^{(2)}(k^2)$ is ultraviolet finite by naïve power counting. We were taught^{[7],[8]} that a closed fermion loop must be regularized as a whole so to preserve gauge invariance. However, having done that, we have affected the finite antisymmetric piece of the vacuum polarization tensor and, consequently, the photon mass. The same reasoning applies when, using Pauli-Villars regularization, we calculate the anomalous magnetic moment of the electron^[7]; again, if care is not taken, we might arrive at a wrong physical result.

In order to get rid of this trouble we should pick out the value of α that just cancels the contribution coming from the regulator fields. From expression (21), we easily find that this occurs for $\alpha = 1/2$, because in this case the signs of the auxiliary masses are opposite, in account of condition (5). We then obtain

$$\Pi^{(2)}(0) = \frac{e^2}{4\pi m} , \quad (24)$$

in agreement with the other approaches already mentioned. We should remember that Pauli Villars regularization violates parity symmetry in 2+1 dimensions^[9]. Nevertheless, for this particular choice of α , this symmetry is restored as the regulator masses get larger and larger.

The result quoted above suggests that the ordinary parity-breaking Pauli-Villars regularization, if carefully implemented, does not introduce any residual contribution to the photon topological mass. In the causal theory of the S-matrix^[10], this corresponds to the minimal splitting of the causal distribution related to the vacuum polarization tensor in QED_3 ^[5].

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